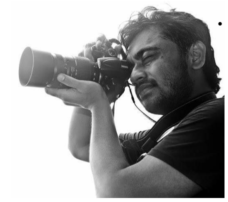
Astrophysics for Fun

Sushan Konar & Sourav Mitra

The Astronomy Module : Lodha Genius Program



Gravitation : *May the force be with you..*

In the larger scheme of things, it is ***Gravitation***, the weakest of the four fundamental forces, which plays the most important role in the evolution of the Universe. For example, the process of stellar evolution can actually be viewed as how *Gravitation* controls the entire sequence of events and with the progress of time plays a more and more dominant role. ***Black Hole***s, the most exotic objects known to mankind, are the extreme cases, where matter entirely looses its identity because of gravity and gets squeezed into a mathematical singularity.

Today's reading material, a 2017 article, alludes to the first ever detection of the *gravitation waves* in 2015 which recorded the merger of two black holes. But the direct observation of a black hole was yet to come. It was in 2019, that humankind *saw* a black hole for the first time, using the event horizon telescope (EHT), at the centre of M87.

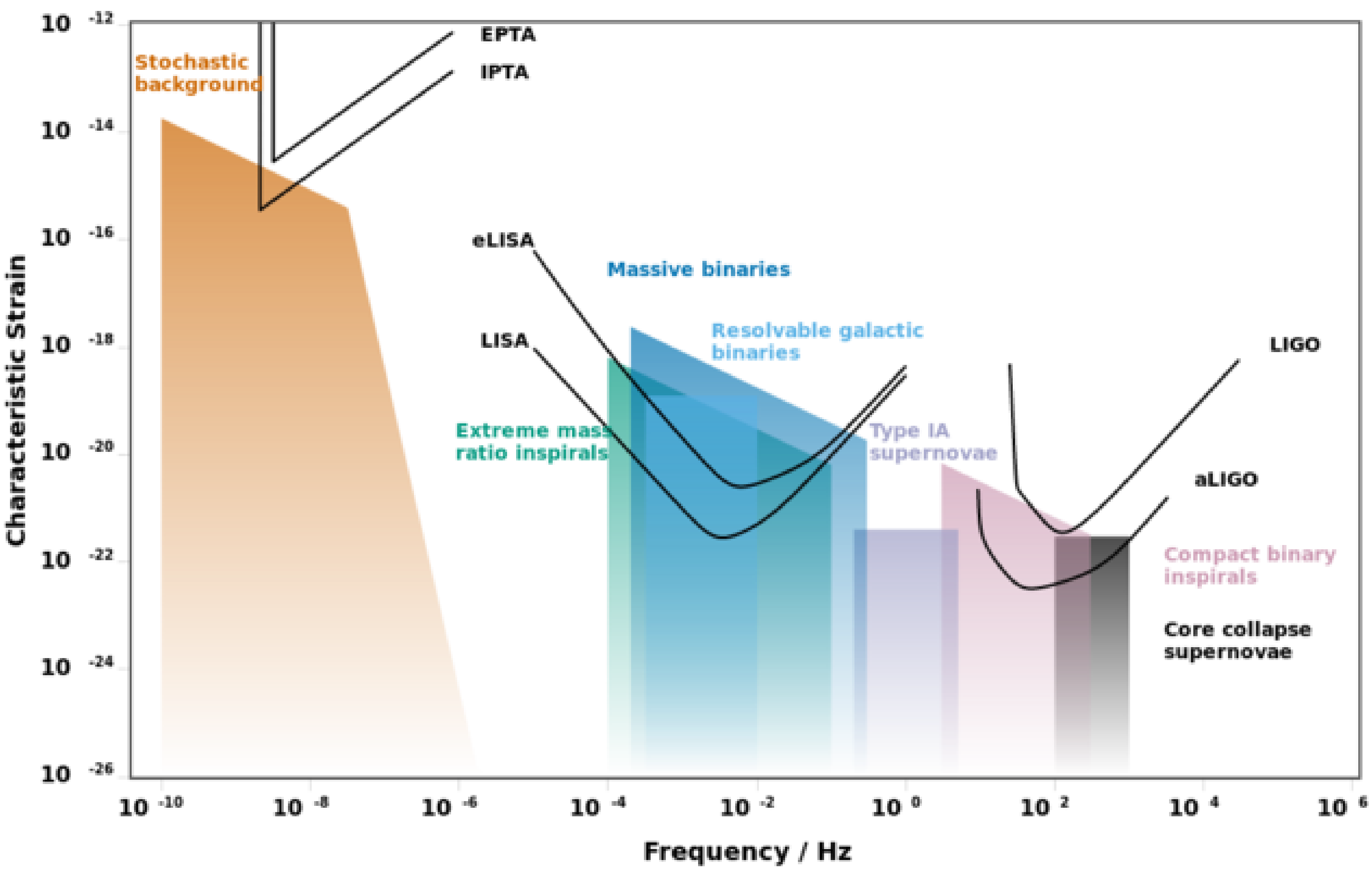
But, *Black Hole* is the endpoint of the story. We must begin at the beginning - when gravitation begins to assert itself over other forces. And it has to do with the height of Mt. Everest, the tallest mountain on planet Earth!

Today, Dr. Mitra would take you through these concepts and show you how every little surface asymmetry (like a mountain) of a self-gravitating, spherical object can give rise to gravitational waves. Whether we are capable of detecting such feeble gravitational waves is another matter altogether.

1. Find the maximum height of a mountain on these rocky planets (moons) - Mercury, Venus, Earth, Mars, Moon (our very own Luna), Ganymede (biggest moon of the Solar System). Assume the rock density to be $\sim 5\ \mathrm{gm.cm^{-3}}$ for all of them.
2. Check if your results match with the actual heights of the tallest mountains on these objects. Do you find any discrepancy? If yes, discuss the possible reasons for that.
3. Calculate the strain produced by the gravitational waves emitted due to the presence of these mountains. Check the detection capabilities of current and proposed detectors from the figure provided. How detectable are the waves produced by these mountains?

Teaching Assistants

Anantharaman S V & Umang Kumar

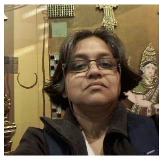


EPTA
IPTA
Stochastic background
eLISA
LISA
Massive binaries
Resolvable galactic binaries
Extreme mass ratio inspirals
Type IA supernovae
LIGO
aLIGO
Compact binary inspirals
Core collapse supernovae
Characteristic Strain
Frequency / Hz
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-24}
10^{-26}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}

Gravity Defied
From potato asteroids to magnetised neutron stars
I. The self-gravitating objects

Sushan Konar

Gravitation, the universal attractive force, acts upon all matter (and radiation) relentlessly. Left to itself, gravity would pull everything together and the Universe would be nothing but a gigantic black hole. Nature throws almost every bit of physics - rotation, magnetic field, heat, quantum effects and so on, at gravity to escape such a fate. In this series of articles we shall explore systems where the eternal pull of gravity has been held off by one or another such means.



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Introduction

It is well known that each and every popular lecture on astrophysics invite questions on black holes, whatever the actual topic of the lecture may be. This abiding interest is simply because black holes are the most exotic of all astrophysical objects, the so called 'unseen' last frontiers. Though we have, at last, been able to 'hear' them - as a pair of black holes merged to form another bigger one. I am, of course, referring to the first ever detection of gravitational waves, generated in two such events creating an unprecedented splash in the entire scientific world last year.

Keywords
fundamental forces, cohesive energy, potato radius, self-gravitating objects

A black hole is a singularity in space-time where matter has been totally defeated by gravity, whereas in everything else matter retains its identity even when gravity dominates (perhaps squeezed into exotic phases, as expected in white dwarfs and neutron stars). However, gravity starts out its journey by being the weakest of all forces, gaining importance only with increasing mass. In this series of articles we shall explore this journey, looking at objects where gravity is either insignificant or can be resisted by another force, and does not win over matter (as it does in black holes). Starting from tiny atoms we shall consider sub-stellar and stellar objects where material property prevails, before such information gets completely lost in the interior of a black hole.

There exist four fundamental forces in the Universe - strong, electromagnetic, weak and gravitational; in the order of their strength of interaction. The strong force, more than two orders of magnitude stronger than the electromagnetic, is effective within a distance scale of $\sim 10^{-13}$ cm and falls off rapidly beyond this. Therefore it has virtually no presence beyond nuclear dimensions. The weak force has an even shorter range - expected to be $\sim 1\%$ of the proton radius. Moreover, both of these forces are effective for only a certain kind of particles. That leaves us with the electromagnetic and the gravitational forces - ones that we usually encounter in our everyday life.

Interestingly, both of these forces are given by the generic form,

$$F = K_F \frac{q_1 q_2}{r^2}, \tag{1}$$

where K_F is a force-specific constant, q_1, q_2 are the corresponding charges and r is the distance between the particles. The forces are directed along r. In cgs units these force equations reduce to,

$$F_{\mathrm{EM}} = \frac{e_1 e_2}{r^2} \quad \text{(electromagnetic)}, \tag{2}$$

$$F_{\mathrm{G}} = G\frac{m_1 m_2}{r^2} \quad \text{(gravitational)}; \tag{3}$$

where e_1, e_2, m_1, m_2 are the charges and masses of the interacting particles and G is the universal gravitational constant. Though omnipresent, the gravitational force is so weak at smaller scales that it can be ignored for all practical purposes. For example, the ratio between these two forces for a pair of protons is,

$$\frac{F_{\mathrm{G}}}{F_{\mathrm{EM}}} = \frac{Gm_p^2}{e_p^2} \simeq 10^{-39}; \tag{4}$$

making the resultant interaction repulsive as the attractive gravitational force is too tiny to make any difference.

The interesting point to note is that there exist electric charges of two different signs whereas there is only one kind of gravitational charge (i.e, mass). This immediately explains why gravity wins at large scales. Any imbalance of charge gives rise to electric fields, causing movement of charges that ultimately neutralises the imbalance. As a result, Universe is charge neutral as a whole and there exist no large scale electric fields. On the other hand, gravity increases with increasing mass. However, other forces give gravity a serious fight at smaller length (mass) scales allowing structures to form. In the following sections we shall consider these structures and see how gravity progressively takes precedence as we move to larger and larger scales.

1. Bound Systems

When a bound system is formed, as a result of an attractive interaction, it has a lower energy than the sum of the energies of its unbound constituents. This difference in energy, released at the time of formation of the bound system, is known as the *binding energy* (E_{B}). There exist different types of E_{B}, operating over different length and energy scales, characteristic of the underlying attractive interaction - the smaller the size of the bound system (for an identical set of constituent particles) the higher being its associated E_{B}.

1.1 *From nuclei to asteroids*

At the smallest scale, strong force binds quarks together into a nucleon (neutron or proton) with E_B in excess of 900 MeV ($E_{\mathrm{B}}^{\mathrm{proton}}$ = 928.9 MeV, $E_{\mathrm{B}}^{\mathrm{neutron}}$ = 927.7 MeV) [1]; whereas nuclear binding energy derives from residual strong force, ranging from 2.2 MeV per nucleon in deuterium to 8.8 MeV per nucleon for Ni^{62}. Further on, at the atomic level, the electron binding energy, arising from the electromagnetic interaction between the electrons and the nucleus, is a measure of the energy required to free an electron from its orbit and is commonly known as the ionisation en-

[1] 1 eV = 1.6 $\times 10^{-12}$ erg

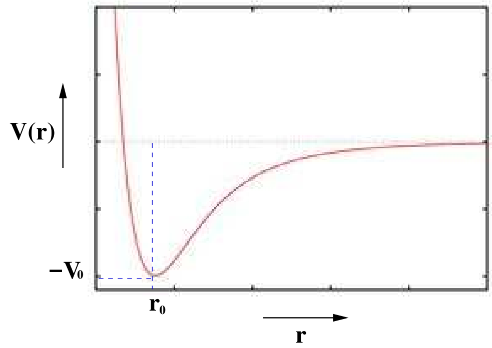

Figure 1. van der Waals potential, $V(r)$, combination of a) a short-range, hard-sphere repulsion and b) a long range attraction as a function of the inter-particle distance r. The resultant reaches its attractive minimum ($-V_0$) at a distance of r_0 (effective distance between two hard-sphere particles touching each other).

ergy (13.6 eV for atomic Hydrogen). For an excellent summary of particle interactions and constituents of bound systems please see the illustration at [2].

The attraction underlying the bond energy of molecules, measuring a few eVs, is again electromagnetic though modified by the electronic structure of a given molecule. At low temperatures, the loosely bound molecules of a gas condense into a liquid/solid phase. The binding energy, known as the cohesive energy, is gained by arranging the molecules into a condensed phase. It is weaker than the intra-molecular bonds, arising out of an effective van der Waals attraction between neutral particles (Fig. 1) and provides a measure for the rigidity. Since the energy required for a substantial deformation of any material must be similar to its binding energy. Therefore, cohesive energy is responsible for holding ordinary solids together - from small chalk pieces in our classrooms to odd-shaped asteroids hurtling through space.

1.2 *Beyond the potato asteroids*

Sometimes art unintentionally catches up with real life. The designers of 'Stars Wars - Episode V', that Hollywood cult classic, actually used some potatoes for the asteroid field scene. Prescient. Recent asteroid exploring missions have observed that while smaller asteroids have irregular 'potato' shapes the larger objects are nearly spherical - the transition happening at an approximate radius of 200-300 km. This is known as the *potato radius* (R_{p}) - separating bona fide asteroids from their more spher-

Eros : 5km x 15km   Phobos : 9km x 13km   Ceres : R ~ 470km

Figure 2. Asteroid shape depends on size - tiny Eros is completely misshapen, while much larger Ceres is almost completely spherical. Pictures taken from `http://solarsystem.nasa.gov/`.

ical counterparts, the dwarf planets.

$R_{\rm p}$ can be obtained from the elastic property of the asteroid material. This is, in fact, related to the maximum height of a mountain on the surface of a solid planet. For an arbitrarily tall mountain, the gravitational pressure at the base would overcome the yield strength of the material and deform the mountain back to the maximum allowable height. The condition for stability of such a mountain (assuming average density of the mountain to be same as that at the base) is that the pressure at the base, given by

$$P_{\rm m} = \rho_{\rm m}\, g_{\rm s}\, h_{\rm m}, \tag{5}$$

is less than the shear stress of the material ($\rho_{\rm m}$ - average mountain density, $g_{\rm s}$ - surface gravity of the planet, $h_{\rm m}$ - mountain height). Hence, the height of the tallest mountain is,

$$h_{\rm m}^{\rm max} = \frac{\sigma}{\rho_{\rm m}\, g_{\rm s}}, \tag{6}$$

where σ is the shear stress of the planetary material. In terms of the planetary radius ($R_{\rm Pl}$) this reduces to,

$$h_{\rm m}^{\rm max} = \frac{3\,\sigma}{4\pi\, G\, \rho_{\rm Pl}\, R_{\rm Pl}}\,; \tag{7}$$

assuming ρ_m to be equal to the average planetary density ($\rho_m \simeq \rho_{\rm Pl} = 3M_p/4\pi R_{\rm Pl}^3$) and the surface gravity ($g_{\rm s} = G\,M_{\rm Pl}/R_{\rm Pl}^2$) to remain constant over the height of the mountain ($\mathrm{M_{Pl}}$ - mass of the planet). For Earth, this formula (using appropriate average density and shear stress) gives a value remarkably close to the height of the Mt. Everest!

An oblong asteroid can be thought of as a small spherical planet plus a large surface mountain. According to Eq.[6], $h_{\rm m}^{\rm max}$ would

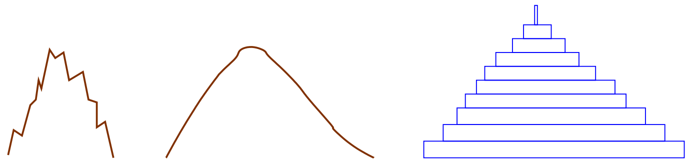

Figure 3. Surface mountains on a rocky planet. While natural mountains (cliff-faced or gently sloping) are bound by the physics discussed in the text, artificial step pyramids may not conform.

decrease with an increase in $R_{\rm Pl}$ and would eventually become smaller than $R_{\rm Pl}$, at which point the asteroid should become approximately spherical. Therefore, the potato radius, $R_{\rm p}$, is attained when the maximum mountain height equals the radius of the asteroid and is given by,

$$R_{\rm p} = \sqrt{\frac{3\sigma}{4\pi G\rho_{\rm Pl}}}\,. \tag{8}$$

Evidently, objects larger than $R_{\rm p}$ are nearly spherical, while smaller objects can have non-spherical shapes because they do not have sufficient gravity to overcome their intrinsic rigidity. This is indicative of the fact that beyond $R_{\rm p}$, gravity dominates over cohesive energy in what are known as the 'self-gravitating' objects.

2. Self-gravitating Objects

A 'self-gravitating' object is defined to be bound by its own gravity - the binding energy coming from the gravitational interaction of its constituents. Earlier, we have seen that gravity dominates over cohesive energy in objects with dimensions larger than $R_{\rm p}$. These larger objects would then be 'self-gravitating'. Let us see if we arrive at the same conclusion from both of these directions.

The gravitational energy, $E_{\rm G}$, of a spherical object of mass M and radius R is approximately given by,

$$E_{\rm G} \simeq \frac{3GM^2}{5R}\,. \tag{9}$$

Let us assume that the cohesive energy of the constituent material is $\epsilon_{\rm c}$ per unit mass. Then the total cohesive energy of the object is

$$E_{\rm c} \simeq \epsilon_{\rm c} M\,. \tag{10}$$

The condition for self-gravitation would then be given by,

$$E_{\mathrm{G}} \gtrsim E_{\mathrm{c}} \Rightarrow R \gtrsim \sqrt{\frac{3}{4\pi G\rho}\epsilon_{\mathrm{c}}}\,. \tag{11}$$

This is exactly equal to R_{p} derived in Eq.[8] - because the cohesive energy, ϵ_{c}, is of the same order of magnitude as the shear stress, σ, of a given solid. In other words, objects with radius larger than R_{p} are self-gravitating. Assuming all the asteroids, dwarf planets and terrestrial planets (we shall talk about Jovian planets later) to be composed of similar rocky material ($\rho \sim 5$ g cm^{-3}, $\epsilon_{\mathrm{c}} \sim 10^9$ cgs units), R_{p} turns out to be in the range of 200-300 Km, exactly the radius at which smallest dwarf planets with spherical shapes have been observed.

Interestingly, gravitaionally bound systems have a rather peculiar property - they have negative specific heat. Consider a small particle of mass m, going around a larger mass M (at rest) in a circular orbit of radius d. Then the balance of force dictates,

$$\frac{GMm}{d^2} = \frac{mV^2}{d} \tag{12}$$

where V is the linear velocity of the mass m. The total energy of the system, given by the sum of its kinetic and potential energy, is

$$E = \frac{1}{2}mV^2 - \frac{GMm}{d} = -\frac{1}{2}\frac{GMm}{d}\,. \tag{13}$$

This is true of every orbit at any arbitrary distance d from the central mass M. Suppose now the velocity of the smaller mass m is increased (by an wish-granting genie, perhaps) to V', where $V' > V$. As a result of this the orbital radius would change to d', where $d' < d$ according to Eq.[12] and the total energy would change to

$$E' = -\frac{1}{2}\frac{GMm}{d'} = -\frac{1}{2}mV'^2 < E\,. \tag{14}$$

Therefore, an increase in the velocity has resulted in an overall decrease in the total energy (and a shrinking of the orbit). If the velocity of an object is considered to be an indicator of its temperature (the rms velocity of the particles of an ideal gas is

Table 1. Escape velocity (V_E), average surface temperature (T_s) and atmospheric compositions of planets in the Solar system.

	planet	V_E (km/s)	T_s (°C)	atmosphere
Terrestrial	Mercury	4	260	
	Venus	10	480	CO_2
	Earth	11	15	N_2, CO_2
	Mars	5	-60	CO_2
Jovian	Jupiter	60	-150	H_2, He
	Saturn	36	-170	H_2, He
	Uranus	21	-200	H_2, CH_4
	Neptune	23	-210	H_2, CH_4
Dwarf	Pluto	1	-220	CH_4

proportional to the temperature of the gas) then an increase in (effective) temperature of the system has resulted in a decrease in the internal energy. Thermodynamically speaking then - a gravitationally bound system has a negative specific heat. (Later, we shall see that the temperature of a gravitationally bound object actually rises as a result of contraction.)

2.1 *Terrestrial vs. Jovian planets*

As far as solid objects are concerned we have already separated them into two classes - with dimensions smaller and larger than the potato radius. The larger objects (all the planets, dwarf planets and satellites) are self-gravitating. However, even among planets there appears to exist another classification - terrestrial and Jovian. Loosely speaking, rocky objects with thin or no atmospheres are termed 'terrestrial' (Earth-like), while large gaseous objects with thick atmospheric covers and icy interiors are known as 'Jovian' (Jupiter-like) planets. Interestingly, the composition of the atmospheres (wherever they exist) also appear to be very different (see table[1]) in these two classes. No marks for guessing that this is nothing but a manifestation of yet another tug-of-war between gravity and some other physical force.

According to the standard theory of solar system formation, both the Sun and the planets formed from a primordial gas cloud. The primary atmospheres of the terrestrial planets as well as those in the Sun and the Jovian planets were quite similar. The composition of this atmosphere is guessed to be $\sim 94\%$ of atomic hydrogen, $\sim 6\%$ of atomic helium and $\sim 0.1\%$ of other gases. However, the terrestrial planets mostly lost this primary atmosphere. This is related to the escape velocity of the planets.

The escape velocity, V_{E}, is the smallest velocity that an object must have to escape from the gravitational attraction of another object. It means that the kinetic energy of the first object should be more than its potential energy in the gravitational field of the second object. Consider a small mass m, in the gravitational field of a large mass M of radius R. For m to escape from the surface of M, it should have a velocity V such that,

$$\frac{1}{2}mV^2 \geq \frac{GMm}{R} \Rightarrow V_{\mathrm{E}} = \sqrt{2GMR}\,. \tag{15}$$

Clearly, larger escape velocities are required to escape from the gravitational field of larger masses (see table[1] for escape velocities from solar system planets).

Evidently, an atmospheric particle would not remain bound to the planet if it happens to have an average velocity that is larger than or equal to the escape velocity of that planet. We know that the rms velocity, V_{rms}, of a gas particle of mass m_g is proportional to the temperature, T, of that gas and is given by

$$m_g V_{\mathrm{rms}}^2 \propto k_{\mathrm{B}}T \Rightarrow V_{\mathrm{rms}} \propto \sqrt{\frac{k_B T}{m}}\,, \tag{16}$$

where k_{B} is the Boltzmann constant. It is to be noted that the lighter particles would have higher V_{rms} for the same temperature. The temperature of a planetary atmosphere is basically determined by the surface temperature of that planet (the layer of gas in contact with the surface would have the same temperature and the temperature would drop exponentially with distance from the surface). Therefore, for large enough surface temperatures and small enough escape velocities (for $V_{\mathrm{rms}} \geq V_{\mathrm{E}}$) the atmospheric particles would escape the gravitational pull of a particular planet.

The inner planets, being closer to the Sun, have higher surface temperatures whereas the outer planets are cooler. Moreover, the inner planets are lighter compared to their Jovian counterparts and therefore have smaller escape velocities. As a result the outer Jovian planets retain their primary atmosphere, whereas in case of the inner terrestrial planets most of the original hydrogen and helium have been lost. In addition, the abundance of hydrogen in cold Jovian planets has allowed it to combine with other elements to form compounds like methane and ammonia. All these factors have been responsible in modifying the atmospheres - resulting in totally different compositions in terrestrial and Jovian planets.

3. Going forward..

Obviously Jovian planets are quite different from their rocky terrestrial counterparts. Instead, they are gaseous and more like colder versions of the Sun (composition-wise) itself. The similarity actually goes deeper and I prefer of think of these objects (in particular, Jupiter and Saturn) as 'could have been stars' rather than planets. Of course, that is a story for another day, to be told in the next article of this series.

Acknowledgment

My thanks to Rajaram Nityananda for bringing Mt. Everest inside our astrophysics class and to Varun Bhalerao for telling me about 'potato' asteroids; also, to the editors of this special 'women only' issue of Resonance for making a 'special' effort.

Suggested Reading

[1] **M. E. Caplan, *Calculating the Potato Radius of Asteroids using the Height of Mt. Everest*, arXiv:1511.04297, 2015**

[2] ***h*ttp://www.physi.uni-heidelberg.de/Einrichtungen/FP/anleitungen/F13/gif/cpep_standardmodel.gif**

[3] **P. A. G. Scheuer, *How high can a mountain be*, JApA, 2, 165-169, 1981**

[4] **P. B. Pal, *A*t the root of things, Resonance, 14(6), 544-567, 2009**